

Theory of graphene and carbon nanotubes

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The purpose of this paper is to give a brief review on characteristic features of electronic states in graphene and its multi-layers mainly from a theoretical point of view. The electron motion in graphene is governed by Weyl's equation for a neutrino or the Dirac equation with vanishing rest mass characterized by velocity v which is about $1/300$ of the light velocity. The pseudo-spin wave function exhibits a sign change due to Berry's phase when the wave vector \mathbf{k} is rotated around the origin and therefore has a topological singularity at $\mathbf{k} = 0$ or at zero energy. This singularity is the origin of the peculiar behavior in the transport properties of graphene, such as the minimum conductivity in the absence of a magnetic field, the quantum Hall effect, the dynamical conductivity [1], and the singular diamagnetic susceptibility [2,3], as well as the absence of backscattering in metallic carbon nanotubes [4–6].

The neutrino equation is invariant under special time-reversal operation S corresponding to the system with strong spin-orbit interaction. Because S is not the real time-reversal symmetry, it can be destroyed by various perturbations, leading to interesting symmetry crossover characteristic to the graphene system, manifesting itself in quantum correction to the conductivity leading to weak and/or anti-localization behavior, conductance fluctuations, the Anderson localization [7–10].

Graphene has often been called a zero-gap semiconductor because density of states $D(E)$ is proportional to the energy. However, graphene is actually a metal instead of semiconductor. In fact, according to the Einstein relation, the conductivity is given by $\sigma = e^2 D^* D(E)$ with diffusion coefficient D^* . Because $D^* \sim v^2 \tau$ with τ being the relaxation time due to impurity scattering and $1/\tau \propto D(E)$, the conductivity becomes independent of energy or carrier concentration, i.e., graphene is a metal, as long as possible dependence of the effective scattering matrix element on the energy is neglected. At zero energy, however, this simple description becomes inappropriate because of the vanishing density of states. Calculations made prior to experiments in a more sophisticated approximation showed that the conductivity takes a universal value of the order of the conductivity quantum $2e^2/\pi^2\hbar$ at zero energy [11]. One most important challenge toward the transistor application is opening up a gap and converting graphene into a semiconductor. There have been several proposals including symmetry breaking between two sublattices using appropriate substrate, graphene ribbons, bilayer graphene in strong perpendicular electric field, etc.

References

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